Midterm 1: Math 20D Midterm Exam 04/26/2023
Instructor: Finley McGlade

You have 50 minutes.

You are permitted the use of a scientific calculator and a double sided page of handwritten notes.

UNLESS THE QUESTION SPECIFICALLY SAYS OTHERWISE YOU MUST SHOW ALL YOUR WORKING.

Name ____

PID ____

"I have adhered to UCSD policies on academic integrity while completing this examination."

Signature ____

The exams consists of 8 pages (including the cover page) with four questions. The maximum possible score is 30+30+30+40=130 points. The first thing you should do when writing time begins is CHECK to make sure you have all 8 of the pages.

Good luck!

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Problem 1. *(30 points) Answer the question below by either writing TRUE or FALSE. No further justification is required. In parts (a) and (b) we make reference to the differential equation*

$$\frac{dy}{dx} = 1 - xy \tag{1}$$

(a) (6 points) True or False. Equation (1) is a linear differential equation.

(b) (6 points) True or False. Equation (1) is a separable differential equation.

(c) (6 points) True or False. The expression $x^2 + y^2 = 8$ *defines an implicit solution to* $\frac{dy}{dx} = \frac{x}{y}$

(d) (6 points) True or False. An integrating factor for the equation

$$\sin(x)y'(x) + 2\cos(x)y = \frac{1}{\sin(x)}e^x.$$

is given by $\mu(x) = \sin^2(x)$.

(e) (6 points) True or False. The functions

$$y_1(t) = \frac{1}{2}(e^t + e^{-t}) \quad \textit{and} \quad y_2(t) = \frac{1}{2}(e^t - e^{-t})$$

are linearly dependent on the domain $\mathbb{R}$.

Problem 2. *(30 points)*

(a) *(15 points) Solve the initial value problem*

$$\frac{dy}{dx} = (1 - y)\cos(x), \qquad y(\pi/2) = 0.$$

(b) *(15 points) Does the differential equation in (a) have a solution satisfying* $y(0) = 1$*. Justify your response.*

Problem 3. *(30 points)*

(a) (15 points) Find constants α, β, C_1, *and* C_2 *such that the initial value problem*

$$y'' - 4y' + 5y = 0, \qquad y(0) = 1/2, \qquad y'(0) = -1/2$$

admits a solution of the form $y(t) = C_1 e^{\alpha t} \cos(\beta t) + C_2 e^{\alpha t} \sin(\beta t)$.

(b) (15 points) Rewrite your solution to (a) in form

$$y(t) = Ae^{\alpha t} \sin(\beta t + \phi)$$

where $A > 0$ *and* $\phi \in [0, 2\pi)$ *are constants.*

Problem 4. *(40 points) At 3:00pm a bottle of wine at* $10^\circ C$ *is placed in an air-conditioned room. Define* $T(t)$ *to be temperature of the bottle of wine* t *minutes after 3:00pm and let* $M(t)$ *denote the temperature of the air-conditioned room* t *minutes after 3:00pm.*

Newton's law of cooling predicts the existence of a ***constant*** $k > 0$ *such that the following differential equation is satisfied*

$$\frac{dT}{dt} = k(M(t) - T(t)).$$

(a) *(20 points) From 3:00pm to 3:20pm the air-conditioning is working and the room is a constant temperature of* $23^\circ C$*. If the wine reachs* $15^\circ C$ *at exactly 3:10pm use Newton's law of cooling to predict the temperature of the wine at 3:20pm. Express your answer to the nearest* $0.01^\circ C$*.*

(b) *(20 points) At 3:20pm the air-conditioning unit in the room loses power. As a result, the temperature of the room changes in such a way that t minutes after 3:00pm, the temperature of the room is given in degrees celsius according to the function*

$$M(t) = \begin{cases} 23, & 0 \leq t < 20, \\ 38 + (23 - 38)e^{-0.05(t-20)}, & t \geq 20. \end{cases}$$

Still assuming that the wine reaches $15^\circ C$ *at exactly 3:10pm, use Newton's law of cooling to predict the temperature of the wine at 3:50pm. Express your answer to the nearest* $0.01^\circ C$.

Scratch Working Page